Scientific Notation

Tuesday, March 30, 2021 4:04 PM

10^{3} - k kilo

10^{6} - M mega

10^{9} - G giga

10^{-3} - m milli

10^{-6} - μ micro

10^{-9} - n nano

10^{-12} - p pico

Circuits, Current, Voltage

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Circuit : A diagram consisting of wires, components, and nodes

Current : Flow of charge in a wire.

Voltage : Potential associated with a node. Can be positive or negative. Only potential difference has physical meaning. Can pick any one node in the circuit as the ground reference.

Note Two nodes connected by a wire has the same voltage.

Note Current stays the same before any branching occurs.

Current (I)

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Def: Current is the flow of positive charge and has unit Amp

Note: although charge is flowing, the material is still charge neutral

Def: If charge $= q$, then current $\boxed{I = \frac{\partial q}{\partial t}}$

Note: Current is directional:

$\underline{\rightarrow 5A}$ is equivalent to $\underline{\leftarrow -5A}$

Voltage (V)

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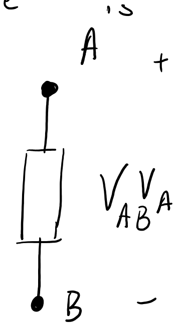
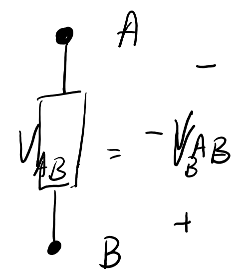
Def Voltage $V = \frac{\partial W}{\partial q}$ and is the electric energy per charge.

where W is energy in joules

q is charge in coulombs

and has unit Volt

Note Voltage is directional:

thus  $V_{AB} = V_A - V_B$ equivalent to  $-V_{AB} = V_B - V_A$

Note Voltage is assumed to be 0 across a wire.

Power (P)

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<u>Def</u> Power is caried by an electric charge.

$$P = \frac{\partial W}{\partial t} = \frac{\partial W}{\partial q} \times \frac{\partial q}{\partial t} = V \times I$$ and has unit <u>Watt</u>

<u>Note</u> The <u>net</u> power for a circuit is 0.

<u>Note</u> Power is <u>directional</u>. We say power is <u>delivered</u> or <u>absorbed</u>.

<u>Passive</u> elements <u>absorb</u> power.

<u>Active</u> elements <u>deliver</u> power.

Example Power Absorption/Delivery

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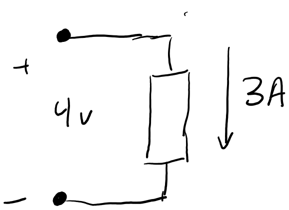
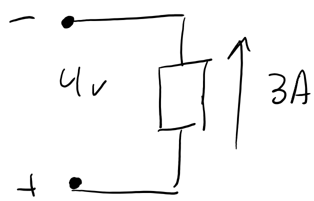
Ex



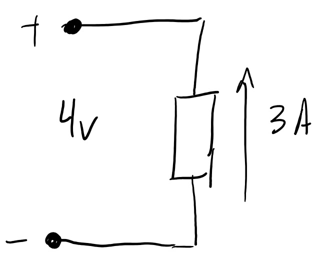
The element absorbs power because the current flows from a high voltage to a lower voltage.

Ex



The element absorbs power.

Ex



The element delivers power. The current flows from a low voltage to a higher voltage.

Elements

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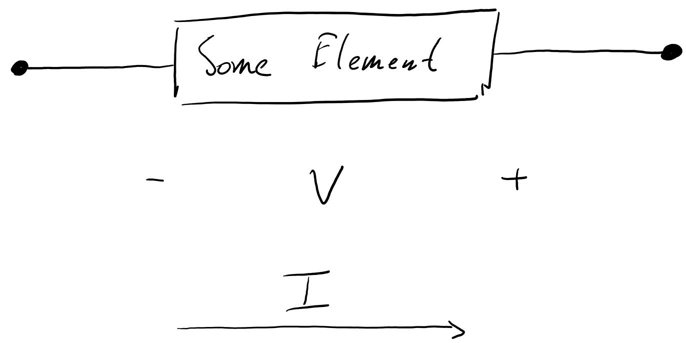
Def circuit elements are defined by a relationship between a voltage difference and current. Different elements have different relationships between thees values.



Voltage Source

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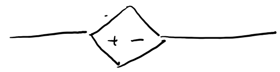
Independent Source

Symbol: 

+ 5V −

Dependent Source

Symbol: 

_ V

Some voltage dependent on the circuit

Currents through voltage sources can be anything.

Current Source

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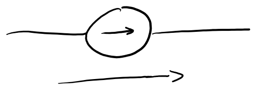
Independent Source

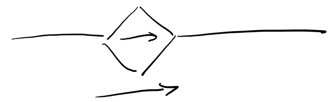
Symbol



$I = 4A$

Dependent Source

Symbol



$I = \_\ A$

current dependent on the circuit.

Voltage across current sources can be anything

Example of Dependent Sources

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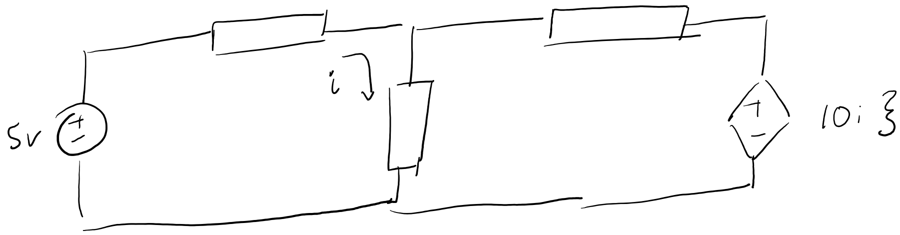
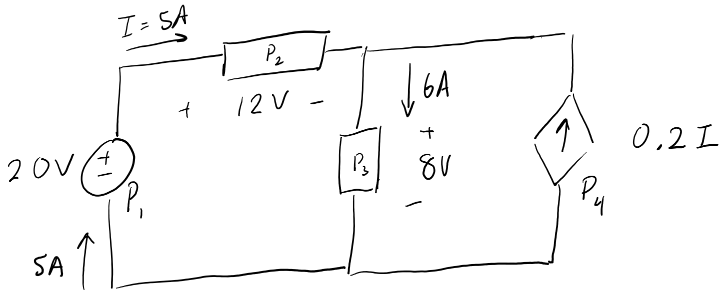


→ the source is $10 * i$, which is the current in A at i. Thus the source provides $10 \frac{V}{A}$.



$P_1 = 20V \cdot 5A = 100W$ supplied

$P_2 = 12V \cdot 5A = 60W$ absorbed

$P_3 = 8V \cdot 6A = 48W$ absorbed

$P_4 = 8V \cdot 1A = 8W$ supplied

Total power supplied: $100W + 8W = 108W$

Total power absorbed: $60W + 48W = 108W$

net power = 0

Resistors

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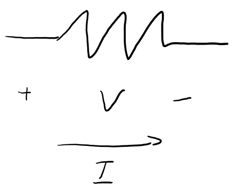
Symbol:



Resistance of a resistor is specified in ohms Ω

Resistors are passive elements, current always flows from (+) → (−)

Power absorbed by a resistor $= V \cdot I = I^2 R = \frac{V^2}{R}$

Ohm's Law

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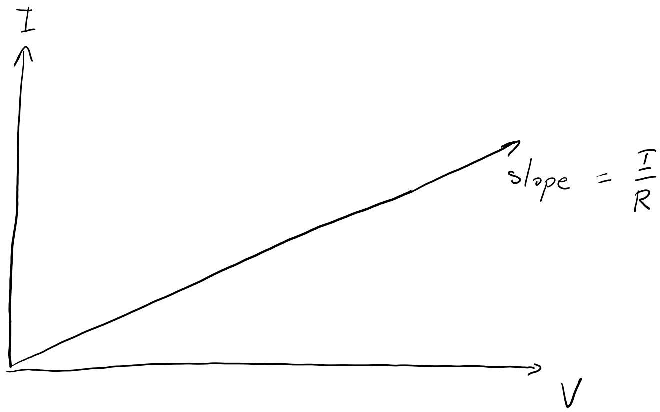
For a resistor: , $V = I * R$ where V is the voltage across the resistor, I is the current, and R is the resistance in ohms. They are signed.



Resistors in Series

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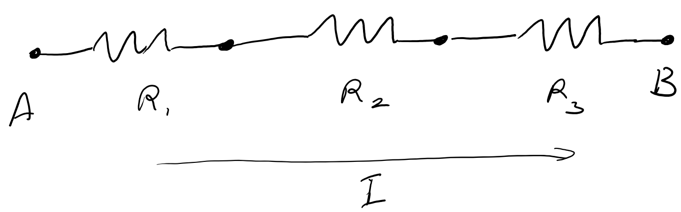
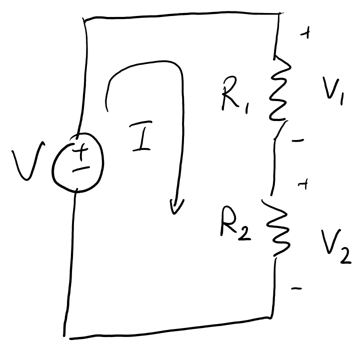
Resistors in Series: must have the same current



$$A - B = R_1 I + R_2 I + R_3 I$$
$$= (R_1 + R_2 + R_3) I$$

Note: The resistance of resistors in series is the sum of resistances.

Voltage Division



$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = I R_1 = V \cdot \frac{R_1}{R_1 + R_2}$$

$$V_2 = I R_2 = V \cdot \frac{R_2}{R_1 + R_2}$$

Resistors in Parallel

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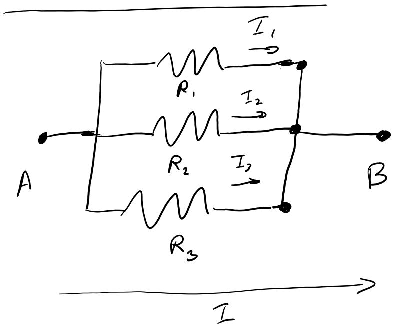
Resistors in Parallel: must have the same voltage



$$I_1 = \frac{A-B}{R_1}$$

$$I_2 = \frac{A-B}{R_2}$$

$$I_3 = \frac{A-B}{3_3}$$

$$I = \frac{A-B}{R_1} + \frac{A-B}{R_2} + \frac{A-B}{R_3}$$

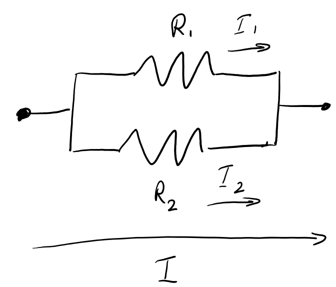
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V$$

since $I = \frac{V}{R}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Note: The resistance of resistors in parallel is the reciprocal of the sum of the reciprocal of resistances.

Current Division



$$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \cdot \frac{R_1}{R_1 + R_2}$$

notice: it is the resistance of other resistor

Voltmeter, Ampmeter

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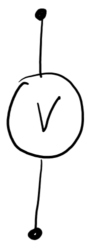
Voltmeter :



measures the voltage between the two points.
has no current that flows through it, so it does not affect the circuit.

Ampmeter :

measures the current flowing through it.
has no voltage drop across it, so it does not affect the circuit.

Open and Short Circuits

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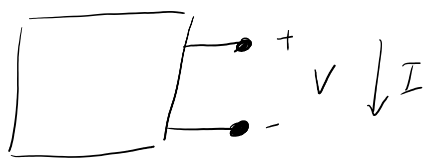
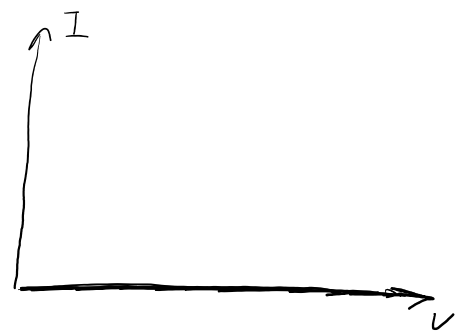
Open circuit: $R = \infty$ $I = 0$ $V =$ anything

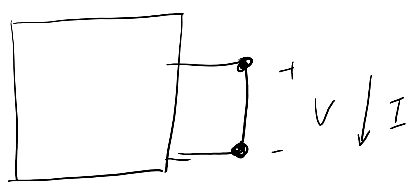


$V =$ anything

$I = 0$

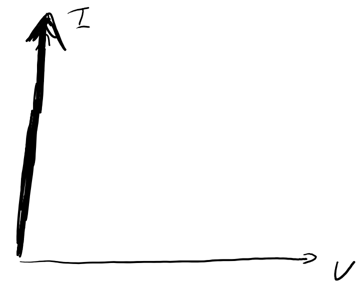


Short circuit: $R = 0$ $I =$ anything $V = 0$



$V = 0$

$I =$ anything



Kirchhoff's Laws

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KCL (Current Law)

The incoming currents of any node must equal the currents leaving the node.

That is, the sum of the individual currents going into a node must equal the sum of the outgoing currents.

KVL (Voltage Law)

The sum of voltage differences in the circuit must equal 0.

Alternatively, starting at some point with voltage A, adding the voltages in a loop following current and arriving back at A, the sum of voltages must equal A.

Solving Circuits

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For a circuit of N components, there are $2N$ unknowns

We can generate N equations from the current-voltage relationship of each component.

Additional equations can be generated by applying KVL and KCL

Node Voltage Method

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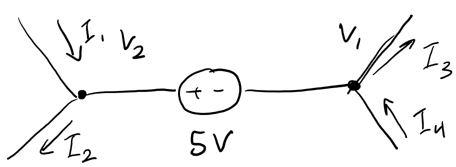
1. Assign voltages to each node given some arbitrary ground
2. Work out the currents in relation to assigned voltages
3. Apply KCL on the nodes and solve for unknowns

- Note that the incoming and outgoing currents must be equal for any <u>groups</u> of elements. We can group these together for KCL and call them supernodes. Supernodes with multiple elements must terminate at the same two points.

- If there are voltage sources, we can pretend they are one "supernode" for KCL, that is we pretend that the voltage source is just a node.

<u>Example</u>:



KCL @ supernode (V_1, V_2):

$$I_1 + I_4 = I_2 + I_3$$

Mesh Current Method

Thursday, April 15, 2021 3:27 PM

1. Assume mesh currents
2. Work out the voltages
3. Apply KVL to meshes

- Note: If there are current sources, we can bypass them by going around them. Since voltages across current sources can be anything, we need to create these "super meshes".

Linear Circuit Theorems

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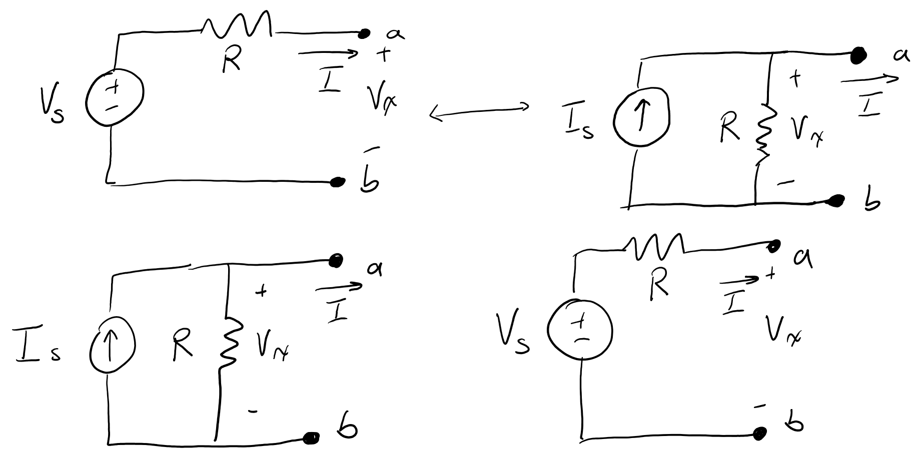
1. Superposition: We can solve a circuit with many sources by solving for only one source at a time. We do this by setting sources to 0, but we cannot set dependent sources to 0.

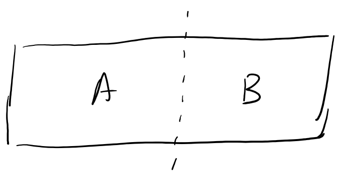
2. Source Transformation: We can convert:

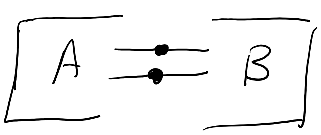


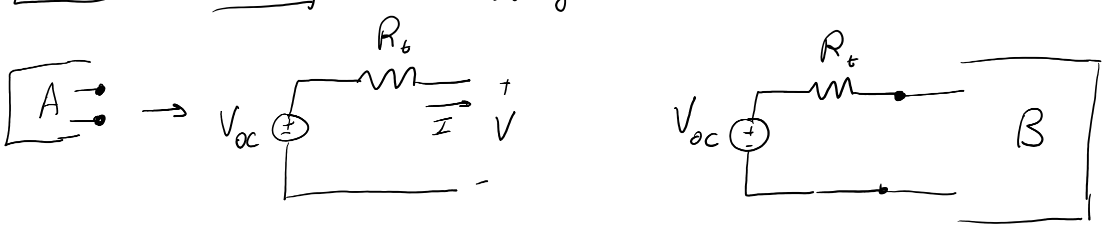
where $I_s = \frac{V_s}{R}$

where $V_s = I_s \cdot R$

3. Thevenin's Theorem: If we can subdivide a circuit into two parts such that each part is connected to the other by exactly 2 wires, then each part can be simplified into a single voltage and resistor in series.







Note: $V_{oc} = V + R_{Th} I$

Solving for Thevenin's Equivalent Circuits

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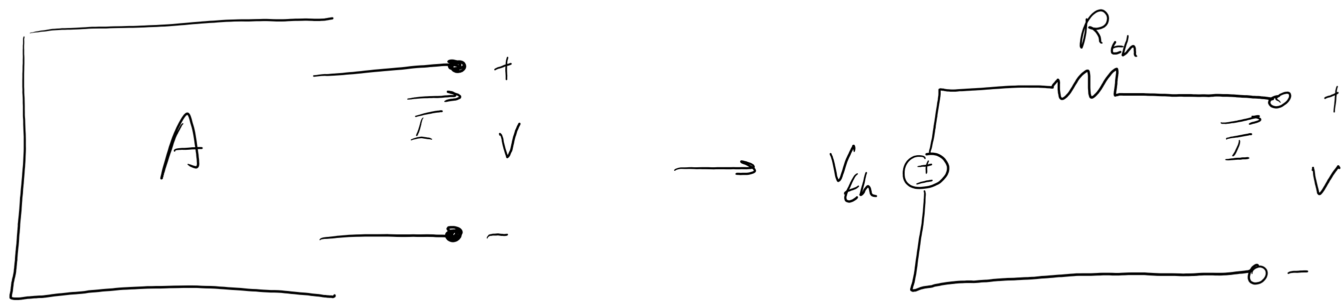

Finding V_{th}, R_{th}:

1) Assume B is an open circuit, solve for V.

$$V_{th} = V_{(oc)}$$

2) Assume B is a short circuit, solve for I.

$$R_{th} = \frac{V_{th}}{I}$$

3) If A has only independent sources, set sources to 0, and solve for the equavelent resistance of A relative to B.

$$R_{th} = R_{A \to B}$$

4) If A has dependent sources, isolate A and keep V and I unknown. Solve for the values using KCL/KVL. Simplify the equations until it becomes:

$$V_{th} = R_{th} I + V$$

Property of Thevenin Equivalent Circuits

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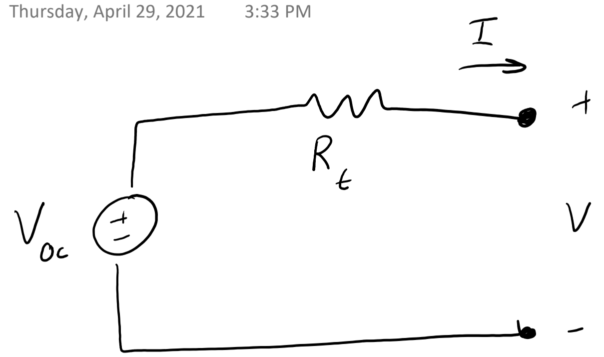
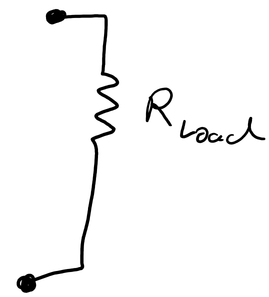
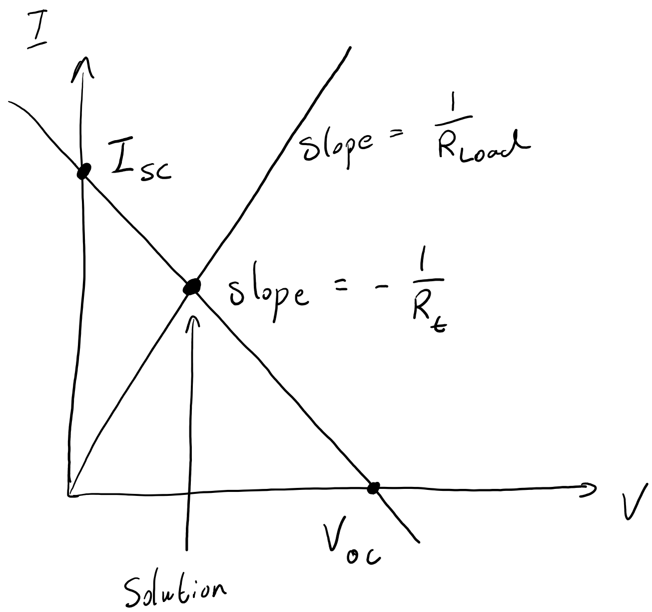


$$V_{oc} = R_t I + V$$





Norton Equivalent Circuits

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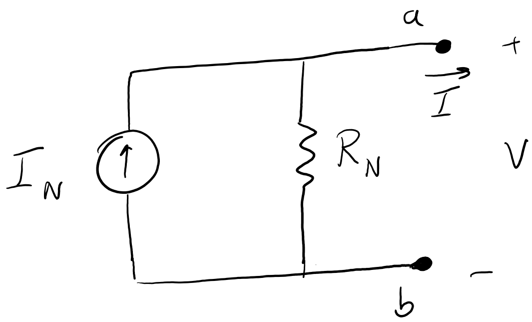
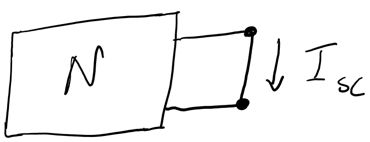


$$I_N = I + \frac{V}{R_N}$$

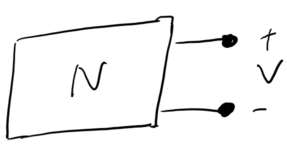
We can use the same methods to solve for Norton equivalent circuits.

1) Short circuit:



$$I_N = I_{sc}$$

2) Open circuit:



$$R_N = \frac{V}{I_N}$$

3) If A has only independent sources

$$R_t = R_{A \to B}$$

4) If A has dependent sources, same as thevenin

Max Power Delivery

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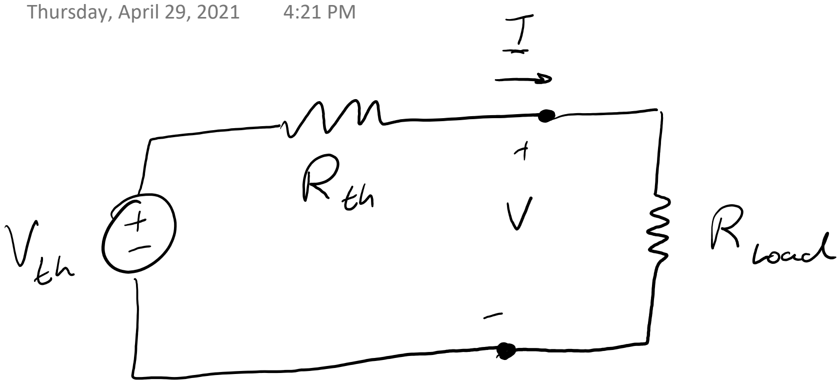


Max power absorbed by R_{Load}:

$$I = \frac{V_{th}}{R_{th} + R_{Load}} \qquad V = I \cdot R_{Load} = \frac{V_{th} R_{Load}}{R_{th} + R_{Load}}$$

$$\text{Power} = V \cdot I = \frac{V_{th}^2 R_{Load}}{(R_{th} + R_{Load})^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{V_{th}^2}{(R_{th} + R_{Load}} \cdot \underbrace{(R_{th} - R_{Load})}_{\text{we want this to be 0.}}$$

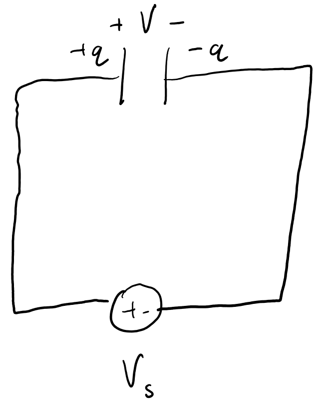
Max power is when $R_L = R_{th}$

max power is $\dfrac{V_{th}^2 R_{th}}{4 R_{th}^2} = \dfrac{V_{th}^2}{4 R_{th}}$

Capacitors

Tuesday, May 4, 2021 3:40 PM

Capacitors are two conductors separated by an insulator. It does not allow DC current to flow, but can produce displacement current.



as $t \to \infty$
$V \to V_s$

and $I = C \cdot \frac{dV}{dt}$

$$V(t) = \frac{1}{C} \int_0^t I(t)\,dt + V(0)$$

Capacitor Properties

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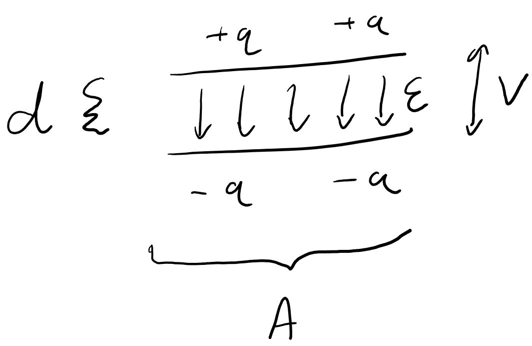


Charge per Area $= \dfrac{q}{A}$

$$\mathcal{E} = \frac{q/A}{\epsilon}$$

$$V = \mathcal{E} \times d = \frac{q}{A} \cdot \frac{d}{\epsilon}$$

$$q = \underbrace{\left(\epsilon \frac{A}{d}\right)}_{\text{capacitance in farads}} V$$

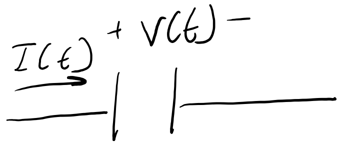
thus $\boxed{q = C \cdot V}$

since $I = \dfrac{\partial q}{\partial t}$, then $\dfrac{\partial q}{\partial t} = C \cdot \dfrac{\partial V}{\partial t}$

and $\boxed{I = C \cdot \dfrac{\partial V}{\partial t}}$

Energy Stored in Capacitor

Tuesday, May 4, 2021 3:55 PM



$$P(t) = V(t) \times I(t)$$

$$E = \int P(t)\, \partial t = \int V \cdot C \cdot \frac{\partial V}{\partial t}\, \partial t = C \int V^2 \partial t$$

$$= \frac{1}{2} C V^2$$

Capacitor Steady State

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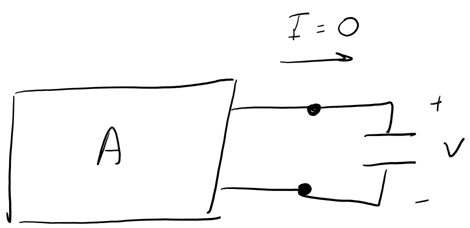
If a capacitor is at steady state, current and voltage are not time dependent. Thus, current must be 0. Voltage will be whatever the equivalent open circuit voltage is.



Capacitors in Parallel

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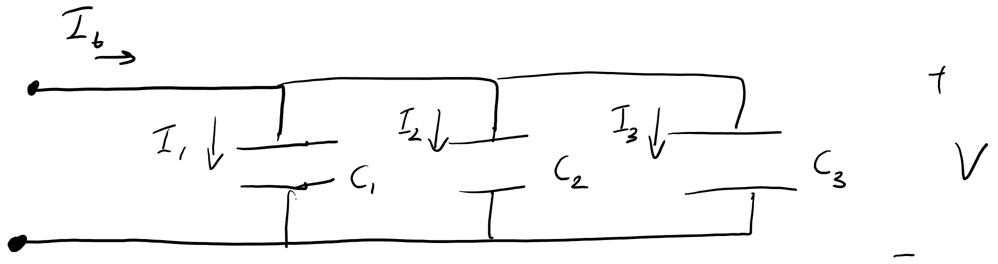

Must have the same voltage across each capacitor

$$I_1 = C_1 \frac{\partial V}{\partial t}$$

$$I_2 = C_2 \frac{\partial V}{\partial t}$$

$$I_3 = C_3 \frac{\partial V}{\partial t}$$

$$I_t = I_1 + I_2 + I_3$$

$$= C_1 \frac{\partial V}{\partial t} + C_2 \frac{\partial V}{\partial t} + C_3 \frac{\partial V}{\partial t}$$

$$C_t = C_1 + C_2 + C_3$$

Capacitors in Series

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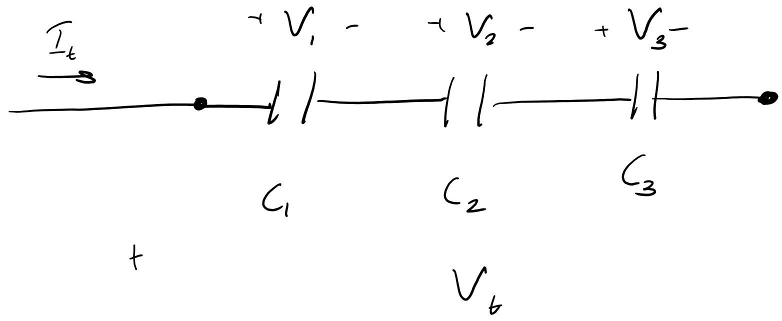


$$\frac{\partial V}{\partial t} = \frac{\partial V_1}{\partial t} + \frac{\partial V_2}{\partial t} + \frac{\partial V_3}{\partial t}$$

$$= \frac{I}{C_1} + \frac{I}{C_2} + \frac{I}{C_3}$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \cdot I$$

$$I = C_1 \frac{\partial V}{\partial t} = C_2 \frac{\partial V}{\partial t} = C_3 \frac{\partial V}{\partial t}$$

Effective capacitance: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Capacitor Voltage over Time

Tuesday, May 11, 2021 3:47 PM

$V(t) = V(0)\, e^{-\frac{t}{RC}}$ for circuits with no independent sources.

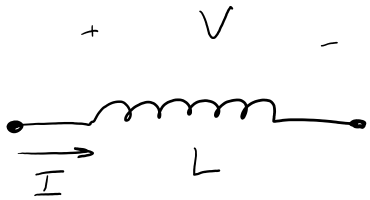
$V(t) = V_s + [V(0) - V_s]\, e^{-\frac{t}{RC}}$ for circuits with independent sources.

$= V(\infty) + [V(0) - V(\infty)]\, e^{-\frac{t}{RC}}$ where $V(\infty)$ is the steady state voltage

Inductors

Tuesday, May 4, 2021 4:33 PM

Symbol:



$$V = L \cdot \frac{\partial I}{\partial t}$$

unit: Henry

$$I(t) = \frac{1}{L} \int_0^t V \, \partial t + I(0)$$

Inductors are coils of wire that store energy with magnetic fields.

Steady state: $V = 0$ (short circuit)

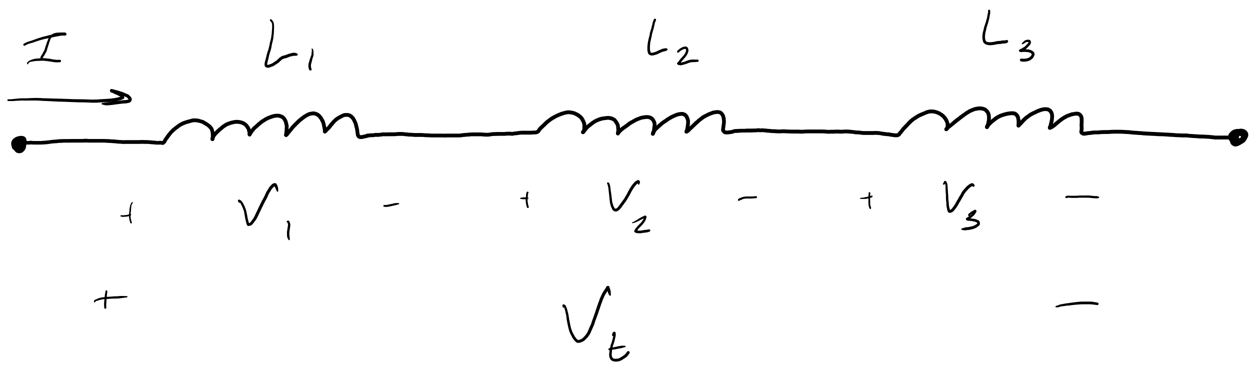
Energy Stored in Inductor

Tuesday, May 4, 2021 4:36 PM

$$E = \int VI\,\partial t = \int L \cdot \frac{\partial I}{\partial t} I\,\partial t = L \int I\,\partial I = \frac{1}{2} L I^2$$

Inductors in Series

Tuesday, May 4, 2021 4:45 PM



$$V_1 = L_1 \frac{\partial I}{\partial t}$$

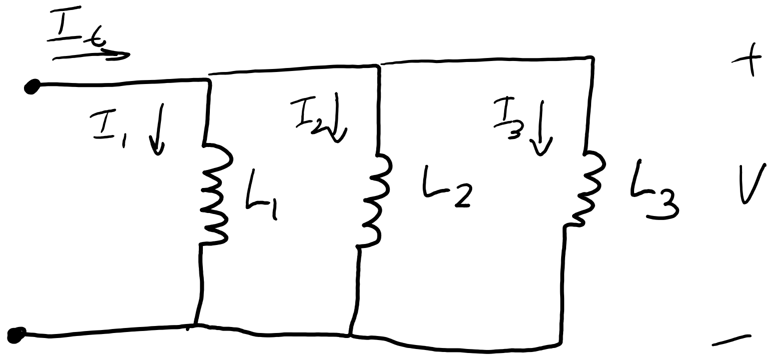
$$V_2 = L_2 \frac{\partial I}{\partial t}$$

$$V_3 = L_3 \frac{\partial I}{\partial t}$$

$$V_t = (L_1 + L_2 + L_3) \frac{\partial I}{\partial t}$$

Inductors in Parallel

Tuesday, May 4, 2021 4:47 PM



$$I_t = I_1 + I_2 + I_3$$

$$V = L_1 \frac{\partial I_1}{\partial t} = L_2 \frac{\partial I_2}{\partial t} = L_3 \frac{\partial I_3}{\partial t}$$

Equivalent inductance: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

Inductor Current over Time

Tuesday, May 11, 2021 3:46 PM

$I(t) = I(0)\, e^{-\frac{Rt}{L}}$ for circuits with no dependent sources.

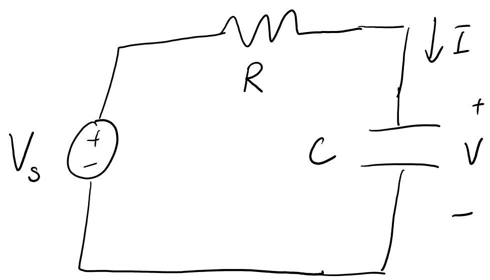
$I(t) = I_s + [I(0) - I_s]\, e^{-\frac{Rt}{L}}$ for circuits with dependent sources

$= I(\infty) + [I(0) - I(\infty)]\, e^{-\frac{Rt}{L}}$ where $I(\infty)$ is steady state current

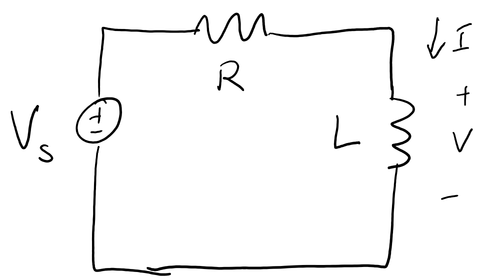
Comparing R,C,L

Thursday, May 6, 2021 3:32 PM

	Resistors	Capacitors	Inductors
	$V = IR$	$I = C \frac{\partial V}{\partial t}$	$V = L \frac{\partial I}{\partial t}$
Stored Energy	none	$\frac{1}{2} C V^2$	$\frac{1}{2} C I^2$
Must be Continous	none	V	I
Steady state	n/a	open circurt	closed circuit



	I	V
$t = 0^-$	0	0
$t = 0^+$	$\frac{V_s}{R}$	0
$t = \infty$	0	V_s

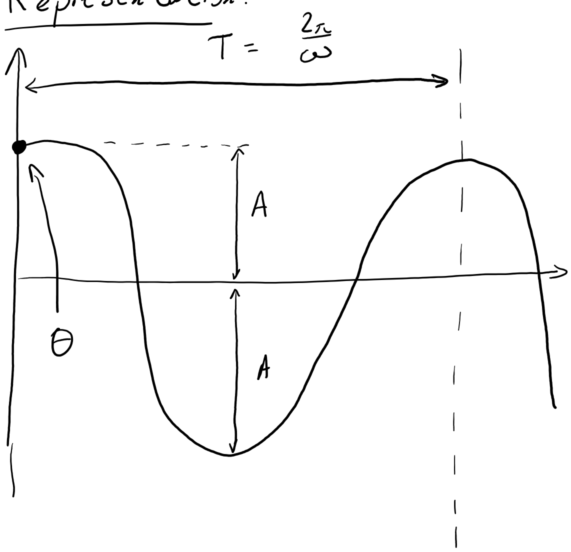


	I	V
$t = 0^-$	0	0
$t = 0^+$	0	V_s
$t = \infty$	$\frac{V_s}{R}$	0

Sinusoidal Circuits

Tuesday, May 18, 2021 3:24 PM

Representation:



$$A \cos(\omega t + \theta)$$

A is the amplitude

ω is the angular frequency,

$T = \frac{2\pi}{\omega} = \frac{1}{f}$ where T is period

θ is the phase (left/right shift)

$$\frac{\partial}{\partial t}\left(A\cos(\omega t + \theta)\right) = \omega A \cos(\omega t + \theta + 90^\circ)$$

Alternatively: $C_1 \cos(\omega t) + C_2 \sin(\omega t)$

$= A \cos(\omega t + \theta)$

$= A \cos(\omega t)\cos(\theta) - A \sin(\omega t)\sin(\theta)$

$C_1 = A\cos(\theta)$

$C_2 = -A\sin(\theta)$

$C_1^2 + C_2^2 = A^2$

$-\tan\theta = \frac{C_2}{C_1}$ $\qquad \theta = -\tan^{-1}\left(\frac{C_2}{C_1}\right)$

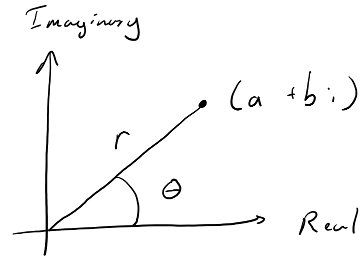
Complex Numbers Review

Tuesday, May 18, 2021 4:14 PM

Complex number representation:

$$a + bi = re^{j\theta}$$

where $i = \sqrt{-1}$



$$a = r\cos\theta$$
$$b = r\sin\theta$$

$$r = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

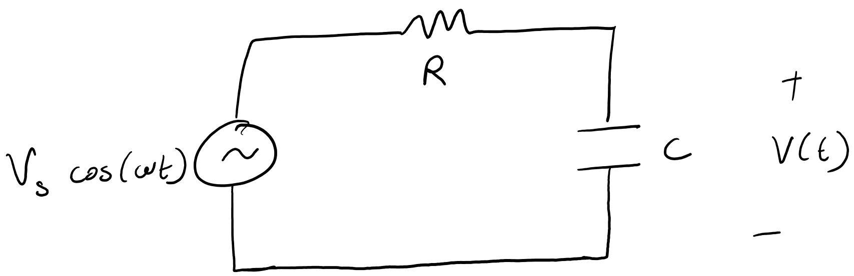
Sinusoidal waves can be written in complex form:

$$A\cos(\omega t + \theta) = Ae^{j(\omega t + \theta)} = Ae^{j\theta}$$ called phasor representation

$$\frac{\partial}{\partial t}(Ae^{j\theta}) = j\omega Ae^{j\theta}$$

Sinusoidal RC Circuit

Tuesday, May 18, 2021 4:27 PM



$$A = \frac{V_s}{1 + (\omega RC)^2}$$

$$B = \frac{V_s(\omega RC)}{1 + (\omega RC)^2}$$

$$V(t) = C \cos(\omega t + \theta)$$

where $C = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1}\left(\frac{B}{A}\right)$

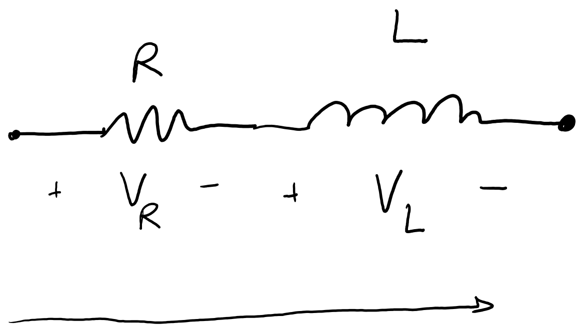
thus:

$$V(t) = \frac{V_s}{\sqrt{1 + (\omega RC)^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{B}{A}\right)\right)$$

$$= \frac{V_s}{\sqrt{1 + (\omega RC)^2}} e^{-j\tan^{-1}(\omega RC)^2}$$

Sinusoidal RL Circuit

Thursday, May 20, 2021 3:37 PM



$$\tilde{I} = Ae^{j\theta}$$

$$V_R = RI$$

$$V_L = L\frac{\partial I}{\partial t} = j\omega L I$$

Impedence

Thursday, May 20, 2021 4:01 PM

Def Impedence is the measure of electrical opposition, Z

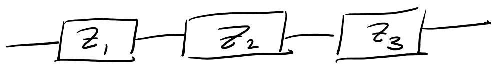
$Z = R + jX$ wher R is the real part, X is the imaginary part.

Resistors: $Z_R = R$

Inductors: $Z_L = j\omega L$

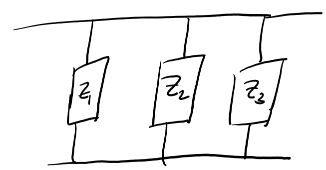
Capacitors: $Z_C = \frac{1}{j\omega C}$

In series:



$Z_{Total} = Z_1 + Z_2 + Z_3$

In parallel:



$$\frac{1}{Z_{Total}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Def $Z = \frac{\tilde{V}}{\tilde{I}}$, $\tilde{V} = \tilde{I} Z$, $\tilde{I} = \frac{\tilde{V}}{Z}$

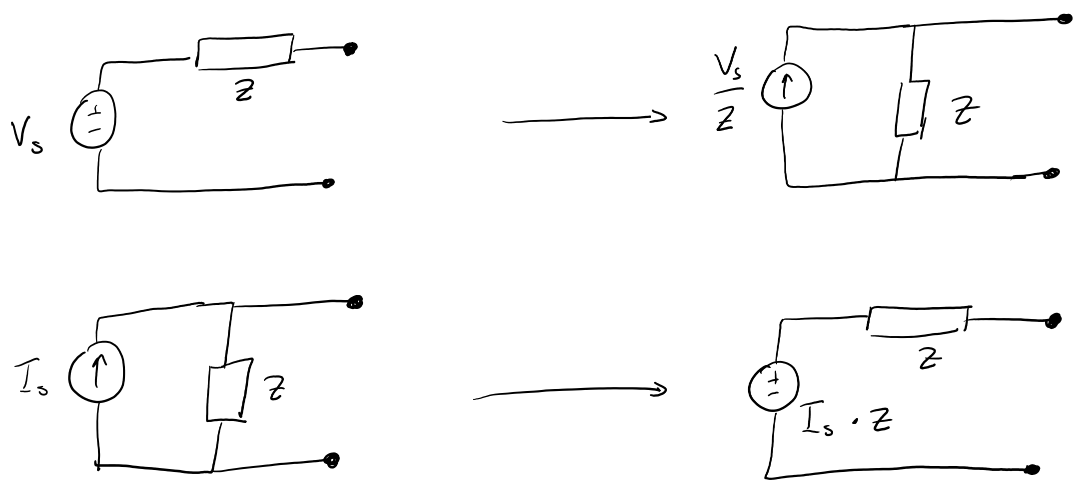
Def Addmittance, $Y = \frac{1}{Z} = \frac{\tilde{I}}{\tilde{V}}$

AC Circuit Theorems

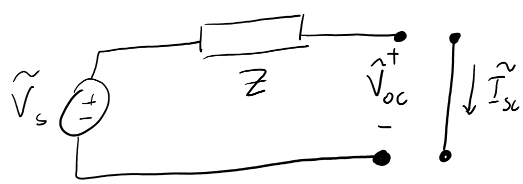
Tuesday, May 25, 2021 4:18 PM

Superposition: We can solve for a mix of DC and AC, or AC with different ω, by solving each source at a time. We can do this by setting all but one source to 0. We can add all the "effects" of every source together to solve.

Source Transformation:



Thevenin's Theorem: Like in DC circuits, any part of a larger circuit can be represented by a Thevenin equivalent circuit.



where $\tilde{V}_s = \tilde{V}_{oc}$

and $Z = \dfrac{\tilde{V}_{oc}}{\tilde{I}_{sc}}$

Lik in DC, we can also find Z by setting sources to 0 and finding the equivalent impedence.

Thus, we can solve for any Thevenin circuit in AC with the same technique as in DC.

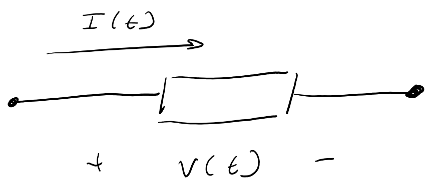
AC Power

Thursday, May 27, 2021 4:42 PM

Given:

$$v(t) = V_s \cos(\omega t + \theta)$$

$$I(t) = I_s \cos(\omega t + \varphi)$$



Then: Element absorbs and supplies power $p(t) = v(t) \cdot I(t)$

<u>Def</u> $P(t) = V(t) \cdot I(t) = V_s \cos(\omega t + \theta) \cdot I_s \cos(\omega t + \varphi)$

$$P_{avg} = \frac{1}{T} \int_0^T P(t)\,dt = \frac{V_s I_s}{T} \cdot \frac{\cos(\theta - \varphi)}{2} \cdot T = \frac{1}{2} V_s I_s \cos(\theta - \varphi)$$

Given:

$$\tilde{V} = V_s e^{j\theta}$$

$$\tilde{I} = I_s e^{j\varphi}$$

<u>Def</u> :

$$\tilde{P} = \tilde{V} \cdot \tilde{I} = V_s I_s e^{j(\theta + \varphi)}$$

$$\tilde{P}_{avg} = \frac{1}{2} V_s I_s \cos(\theta - \varphi) = \frac{1}{2} Re\left[\tilde{V}_s \cdot \tilde{I}_s\right]$$

Average Power Supplied by R, L, C

Tuesday, June 1, 2021 3:36 PM

Resistor: $P = \frac{1}{2} Re\left[\tilde{V} \cdot \hat{I} \right] = \frac{1}{2} Re\left[R \cdot \hat{I} \cdot \tilde{I} \right]$ $\quad P_{avg} = \frac{1}{2} R |\tilde{I}|^2$

Inductor: $P = \frac{1}{2} Re\left[\tilde{V} \cdot \hat{I} \right] = \frac{1}{2} Re\left[j\omega L \tilde{I} \cdot \hat{I} \right]$ $\quad P_{avg} = 0$

Capacitor: $P = \frac{1}{2} Re\left[\tilde{V} \cdot \tilde{I} \right] = \frac{1}{2} Re\left[\frac{\hat{I} \cdot \tilde{I}}{j\omega L} \right]$ $\quad P_{avg} = 0$